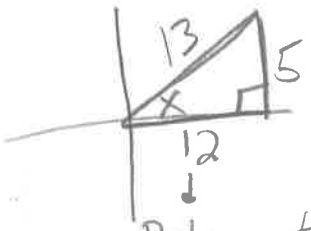
Key

Math 4
4-8 Practice

Name_____
Date_____

1st Q

1. *Given*: $\sin x = \frac{5}{13}$, $0 < x < \frac{\pi}{2}$ **Find: sin 2x, cos 2x, tan 2x. In what quadrant does 2x lie?**



Pythag. Theorem

$\sin x = \frac{5}{13}$

$\cos x = \frac{12}{13}$

$\tan x = \frac{5}{12}$

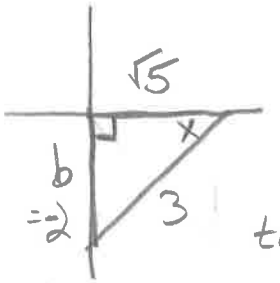
$\sin 2x = 2\left(\frac{5}{13}\right)\left(\frac{12}{13}\right) = \frac{120}{169}$

$\cos 2x = \left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2 = \frac{144}{169} - \frac{25}{169} = \frac{119}{169}$

$\tan 2x = \frac{\sin 2x}{\cos 2x} = \frac{120}{169} \cdot \frac{169}{119} = \frac{120}{119}$

1st quadrant

2. *Given*: $\cos x = \frac{\sqrt{5}}{3}$, $\frac{3\pi}{2} < x < 2\pi$ **Find: sin 2x, cos 2x, tan 2x. In what quadrant does 2x lie?** sin + cos positive



$\cos x = \frac{\sqrt{5}}{3}$

$\sin x = \frac{-2}{3}$

$\tan x = \frac{-2}{\sqrt{5}}$

$(\sqrt{5})^2 + b^2 = 3^2$

$b^2 = 4$

$\sin 2x = 2\left(\frac{-2}{3}\right)\left(\frac{\sqrt{5}}{3}\right) = \frac{-4\sqrt{5}}{9}$

$\cos 2x = \left(\frac{\sqrt{5}}{3}\right)^2 - \left(\frac{-2}{3}\right)^2 = \frac{5}{9} - \frac{4}{9} = \frac{1}{9}$

$\tan 2x = \frac{\frac{-4\sqrt{5}}{9}}{\frac{1}{9}} = \frac{-4\sqrt{5}}{9} \cdot \frac{9}{1} = -4\sqrt{5}$

4th quadrant

sin is neg & cos is pos.

3. Solve for primary values: $\sin 2x + \cos x = 0$

$2\sin x \cdot \cos x + \cos x = 0$

$\cos x (2\sin x + 1) = 0$

$\cos x = 0$: $x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\sin x = \frac{-1}{2}$: $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

4. Solve for primary values: $1 - 3\cos x - \cos 2x = 0$

$\cos 2x = 2\cos^2 x - 1$

$1 - 3\cos x - (2\cos^2 x - 1) = 0$

$-2\cos^2 x + 1 + 1 - 3\cos x = 0$

$0 = 2\cos^2 x + 3\cos x - 2$

$0 = (2\cos x - 1)(\cos x + 2)$

$\cos x = \frac{1}{2}$: $x = \frac{\pi}{3}, \frac{5\pi}{3}$

$\cos x = -2$: No solutions

5. Solve for primary values: $\sin 2x = -\frac{\sqrt{3}}{2}$

$U = 2x$

$\sin U = -\frac{\sqrt{3}}{2}$

$U = \frac{4\pi}{3}, \frac{5\pi}{3}$

$2x = \frac{4\pi}{3}$

$x = \frac{4\pi}{6}$

$= \boxed{\frac{2\pi}{3}}$

$= \frac{2\pi}{3} + \pi$

$= \boxed{\frac{5\pi}{3}}$

$2x = \frac{5\pi}{3}$

$x = \boxed{\frac{5\pi}{6}}$

$\frac{5\pi}{6} + \pi$

$= \boxed{\frac{11\pi}{6}}$

7. Verify: $\frac{1+\cos 2x}{\sin 2x} = \cot x$

$\frac{1 + \cos^2 x - 1}{\sin x \cdot \cos x}$

$= \frac{\cos x}{\sin x}$

$= \boxed{\cot x}$

8. Verify: $\sin 2\alpha = \frac{2\tan\alpha}{1+\tan^2\alpha}$

$= \frac{2\tan\alpha}{\sec^2\alpha}$

$= \frac{\frac{2\sin\alpha}{\cos\alpha}}{\frac{1}{\cos^2\alpha}}$

$= \frac{2\sin\alpha}{\cos\alpha} \cdot \frac{\cos^2\alpha}{1}$

$= 2\sin\alpha \cdot \cos\alpha$

$= \boxed{\sin 2\alpha}$